THE ASTROPHYSICAL JOURNAL, 294:L125–L129, 1985 July 15

TURBULENCE IN MOLECULAR CLOUDS: A NEW DIAGNOSTIC TOOL TO PROBE THEIR ORIGIN

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Received 1985 February 15; accepted 1985 May 9

ABSTRACT

We present a method that utilizes the observational relation between velocities and sizes of molecular clouds (MC), together with a recent model for large-scale turbulence, to uncover the instability responsible for the type of turbulence observed in MC and the value of the physical parameters of the "placental medium" from which turbulence originated.

Subject headings: interstellar: matter — turbulence

I. THE PROBLEM

A comprehensive analysis of the measured velocities and cloud sizes showing a significant correlation of the type

$$v(l) = v_0\left(\frac{l}{l_0}\right)^m, \qquad 0.45 \le m \le .70, \tag{1}$$

led Larson (1981) to suggest that molecular clouds (MC) are in a turbulent state (see also Martin and Barrett 1978; Solomon, Scoville, and Sanders 1979; Fleck 1983). Relation (1) has been found to hold in three *broad* regions (Falgarone and Puget 1984), with radii (in pc), masses (in $M_\odot$), densities (in cm^{-3}), and velocities (in $\mathrm{km\ s}^{-1}$) given by

$$\text{A:} \quad R \approx 50, \quad M \approx 3.10^5, \quad n \approx 12, \quad v \approx 6. \tag{2a}$$

This corresponds to the giant molecular clouds (GMC), often referred to as CO clouds (Dame *et al.* 1984).

$$\text{B:} \quad R \approx 2, \quad M \approx 150, \quad n \approx 750, \quad v \approx 2. \tag{2b}$$

This corresponds to isolated dark clouds (Wilking and Lada 1983; Leung, Kutner, and Mead 1982; Blitz 1980; Young *et al.* 1982), and finally

$$\text{C:} \quad R \approx 0.2, \quad M \approx 3, \quad n \approx 2000, \quad v \approx 0.3, \tag{2c}$$

which comprises the smallest dense cores (Dickman and Clemens 1983; Snell *et al.* 1984; Myers, Lince, and Benson 1983; Perault, Falgarone and Puget 1984).

The validity of relation (1) for these independent sets of clouds, ranging over three decades in cloud sizes, suggested to Myers (1983) that a "widespread and fundamental process," rather than a simple coincidence, must be at work.

Stated differently, is there a way to use relation (1) to uncover the physical instability responsible for the kind of turbulence observed in MC?

The goal of this *Letter* is to propose a new method to extract from the observational relation (1) the following: (*a*) the instability responsible for turbulence, and (*b*) the value of the parameters of the "placental medium" out of which turbulence originated. Traditionally, the study of the formation of large structures out of an otherwise stable homogeneous medium begins by assuming a given instability (Parker 1966, 1967*a*, *b*, 1979; Lerche 1967; Elmegreen 1982). The corresponding growth rate $n(k)$ versus k is derived and then maximized so as to obtain the fastest growing mode, to which there corresponds a given wavelength. If the latter quantities compare favorably with the typical time scales and sizes of the system, the instability is assumed to be actually at work.

The above procedure has two limitations. First, one can never be certain that a given instability, however plausible, is the only one at work. Second and most important, the linear mode analysis cannot fulfill the ultimate goal of a complete instability analysis, namely that of describing what *happens* to the instability which in the final stages is responsible for the break-up of turbulence. To do so, one needs a model of turbulence to describe how the energy pumped into the largest scales of the system by the instability gets distributed to all the smaller scales. Since this cascading process is caused by the nonlinear interactions (largest eddies become unstable and break up into smaller units), it is clear that one cannot elucidate the observationally most interesting phase by means of the linear growth rate analysis. One needs a model for large-scale turbulence, LST.

Recently, an analytical model for LST has been constructed (Canuto and Goldman 1985, hereafter CG), and the results for convective turbulence have been favorably compared with laboratory and astrophysical data. The model, based on a new expression for the nonlinear interactions, yields an analytical solution for the nonlinear equation satisfied by the turbulence spectral function, $F(k)$ (see eq. [6]). Since the growth rate $n(k)$ enters as an ingredient, we can formally write (see eq. [7])

$$F(k) = F[n(k)]. \tag{3}$$

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In laboratory turbulence, one knows the form of the instability $n(k)$ that generates turbulence. One can then use equation (3) to obtain the spectral function $F(k)$ and other quantities therefrom.

In astrophysics, one never knows for certain which instability is at work. However, one knows, for example, equation (1) which is equivalent to knowing $F(k)$. One can therefore invert equation (3) and obtain

$$n(k) = n[F(k)]. \tag{4}$$

This method of retrieval of the growth rate can be applied to any of the three regions discussed above (or to turbulence other than that encountered in MC). However, we shall limit ourselves to region A. In fact, since GMC supposedly originate from an instability of the interstellar medium, whose basic properties are reasonably well known, our retrieval method can be tested in region A more stringently than in regions B and C, whose placental medium is less accurately known.

II. TURBULENCE

Turbulence is the result of disturbances, i.e., instabilities generated in an otherwise laminar flow. The resulting flow is characterized by a spectrum of eddies that can conveniently be divided into two broad intervals:

Low wavenumber region.—This region is populated by large eddies containing most of the energy and whose behavior depends critically on the characteristics of the instability generating the turbulent state. Because of the almost unlimited number of ways in which an instability can be generated, large-scale turbulence cannot be expected to exhibit a "universal spectrum," and theoretical treatments of this part of the eddies' spectrum have therefore been considered extremely difficult.

High wavenumber region.—Because of the difficulties in describing the preceding region, Heisenberg and Kolmogoroff (HK) proposed a model valid for those eddies that are sufficiently removed from the low wavenumber region to behave independently of the specific features of the stirring mechanism (i.e., the instability) and also sufficiently removed from the region where (kinematic) viscosity operates. Having been selected so as to be disconnected from both the source and the sink of energy, the HK eddies exhibit a universal character. Their $v(l)$ versus l relation is found to be (Batchelor 1970)

$$v(l) \sim l^{1/3}, \tag{5}$$

which is known as the Kolmogoroff spectrum. Since the HK eddies no longer contain the imprint of the stirring mechanism, they do not allow the retrieval of the underlying instability. *If* MC satisfied expression (5), there would be no hope of identifying the physical process(es) that generated the turbulence we observe. However, comparison of relations (1) and (5) shows that the turbulence found in MC is not of the Kolmogoroff type, a conclusion already arrived at by Scalo (1984). Therefore, *if* relation (1) is interpreted in terms of a turbulent energy cascade (see, however, Henriksen and Turner 1984), we deal with LST which allows us to carry out the retrieval of the "placental instability."

III. LARGE-SCALE TURBULENCE

Since turbulent energy is distributed among eddies of different sizes, one defines a spectral function $F(k)$.

$$v^2(k) = \int_k^\infty F(k)\, dk. \tag{6}$$

To obtain the total turbulent energy, one must integrate from $k = k_0$, where $l_0 = \pi/k_0$ is the size of the largest eddy. For the HK eddies, it is well known that $F_{\rm HK}(k) \sim k^{-5/3}$ which in turn yields the $v(l)$ versus l relation (5).

For the large energy-containing eddies, the model proposed by Canuto and Goldman (CG) yields the spectral function $F(k)$ in terms of the growth rate $n(k)$ of the underlying instability. The result is ($' = d/dk$)

$$-2\gamma F(k) k^2 = \left[k n^{1/2} \int_{k_0}^k k n^{1/2} (n k^{-2})'\, dk \right]', \tag{7}$$

where γ is numerical constant provided by the theory itself (see eq. [6b] of CG).

IV. RETRIEVAL METHOD

Once a $v(l)$ versus l function is known, one can eliminate $F(k)$ between equations (6) and (7). Introducing the variables

$$v(k) = v_0 V(x), \qquad x = k/k_0, \qquad k = \pi/l,$$
$$n(k) = n_0 f(x), \qquad n_0 = \gamma^{1/2} k_0 v_0, \tag{8}$$

the result is an equation for $f(x)$, i.e.,

$$[g(x) f^2 - x^2 g^2](df/dx) - A(x) f^3 + B(x) f = 0, \tag{9}$$

where ($' = d/dx$),

$$g(x) = -2x^{-2} \int_1^x V(x) V'(x) x^2\, dx,$$
$$A(x) = 2g(x) x^{-1}, \qquad B(x) = 2xg(xg)'. \tag{10}$$

Using the form

$$V(x) = x^{-m}, \qquad 0.45 \le m \le 0.70 \tag{11}$$

suggested by the data, we solved equation (9) numerically. (We were unable to find an analytic solution.) It can be verified that at the point

$$x_* = m^{1/(2m-2)}, \tag{12}$$

$$f(x_*) = 1, \qquad f'(x_*) = \frac{3}{2x_*}\left[1 - \frac{1}{3}(1 + 16m)^{1/2}\right],$$
$$g(x_*) = m^{1/(1-m)}, \qquad g'(x_*) = 0. \tag{13}$$

The resulting growth rate $n(k)/n_0$ versus k/k_0 is plotted in Figure 1 for $m = 0.45, 0.5$, and 0.70. In particular, $m = 1/2$ yields

$$f(x) = 1, \qquad \text{i.e., } n(k) = n_0 = \text{constant}. \tag{14}$$

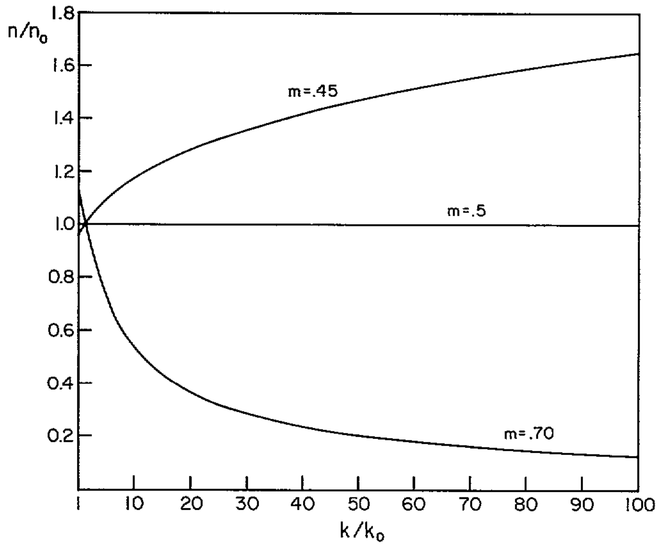

FIG. 1.—The growth rate $n(k)$ vs. k extracted from the observational $v(l)$ vs. l relation, eq. (1). The three curves correspond to three values of the parameter "m." The normalizations n_0 and k_0 are given in terms of the observational l_0 and v_0 in eq. (8).

Since Figure 1 is a direct consequence of the observational $v(l)$ versus l relation, it represents a *universal diagnostic diagram* against which one can compare the expressions of known instabilities so as to single out the one responsible for the turbulence observed in MC.

V. AN ILLUSTRATIVE EXAMPLE

Among the instabilities frequently considered for the interstellar medium is the Rayleigh-Taylor instability. We shall show how Figure 1 suggests that a magnetic field must be present. In fact, the growth rate corresponding to a nonmagnetic R-T instability diverges for $k \to \infty$ as

$$n(k) \sim k^{1/2} \quad \text{or} \quad n(k) \sim k,$$

which are incompatible with the behavior of the $n(k)$ of Figure 1. This divergent behavior is, however, known to disappear if a magnetic field is present, in which case $n(k)$ becomes constant as $k \to \infty$, and thus compatible with Figure 1. This qualitative argument illustrates how Figure 1 can be used as a tool to discriminate among possible instabilities.

Let us now analyze the magnetic R-T instability in more detail. A linear mode analysis (Parker 1979) of a medium characterized by ($\boldsymbol{B} = \hat{y}B$)

$$\alpha = \frac{B^2}{8\pi\rho u^2}, \quad g\Lambda = (1+\alpha)u^2, \quad p = p_0 e^{-z/\Lambda} \quad (15)$$

indicates *that the gas falls into condensed clouds suspended into the magnetic field* in the $y - z$ plane (Parker 1966, 1967*a*, *b*, 1979). Elmegreen (1982) has extended Parker's analysis to include self-gravity and has concluded that this process may indeed lead to the formation of giant cloud complexes. However, once that has occurred, *there is still a tendency of the gas to break up into smaller dimensions across the magnetic field, i.e., in the x-direction.* Parker (1967*a*, *b*) showed that the latter instability occurs "over so broad a spectrum of k_x as to produce motions which have some resemblance to white noise or turbulence." While Parker could not carry the analysis any further, his $n(k)$ versus k_x (Parker's Fig. 1) may fit the allowed region in Figure 1 provided the parameters α, u, and B are properly chosen. Since Parker normalizes growth rate and wavenumbers using Λ/u and Λ, ($\Omega \equiv n\Lambda/u$, $q \equiv \Lambda k$), we have

$$n(k)/n_0 = R_1\Omega, \qquad k/k_0 = R_2 q,$$

$$R_1 = \frac{l_0}{\Lambda}\frac{u}{v_0}\left(\frac{1}{\gamma\pi^2}\right)^{1/2}, \qquad R_2 = \frac{1}{\pi}\frac{l_0}{\Lambda}. \quad (16)$$

Using the second expression of (15) and imposing that $n(k)/n_0 = 1$, corresponding to $m = 1/2$ in equation (1), we have

$$\Omega^2 = b(1+\alpha) = \text{const}, \qquad b = \gamma\pi^2\frac{\Lambda v_0^2}{g l_0^2}. \quad (17)$$

Fitting the data by Dame *et al.* (1984) to equation (1) with $m = 1/2$ yields a value for v_0^2/l_0 which once substituted in expression (17), together with $\gamma = (3\pi/8)^2$, finally gives

$$b = 10\left(\frac{\Lambda}{l_0}\right)\frac{1}{g_9}, \quad (18)$$

where g_9 is the local gravity in units of 3.5×10^{-9} cm s^{-2} (Elmegreen 1982). Since the size of the system l_0 is somewhat arbitrary, we shall define $l_0 = 2z_*$, where $\rho(z_*)/\rho_0 = 0.10$. We then have $4.6\ \Lambda = l_0$, i.e., $b \approx 2$. Parker's dispersion relation for Ω then becomes an equation for α (the other two quantities q_2 and q_3 can be taken from Elmegreen's analysis). Solving for α, we obtain the results shown in Table 1. No solution exists for $\Gamma > 0.33$ (Myers 1978 has $\Gamma = 0.25$).

The values in the table are quite close to those generally accepted for the intercloud gas (Spitzer 1978; Mouschovias 1976; Mouschovias, Shu, and Woodward 1974). It is important to remark that these values result from the use of a single observational input, namely equation (1).

TABLE 1

PROPERTIES OF THE "PLACENTAL MEDIUM" OBTAINED FROM THE RETRIEVAL METHOD

Γ	α	u	$B/(n\mu)^{1/2}$
0.10.......	0.14	1.23	2.3
0.20.......	0.07	1.27	1.7
0.25.......	0.04	1.29	1.3
0.30.......	0.02	1.30	.83

NOTE.—Γ is the adiabatic index of the perturbation. u is in units of 10 km s^{-1}. B is in μG. n is in cm^{-3}. μ is the mean molecular weight in units of 10^{-24} g.

VI. CONCLUSIONS AND CAVEATS

The ever-increasing role played by turbulence in astrophysical settings is in stark contrast to the lack of a reliable theory to treat it. Heisenberg and Kolmogoroff succeeded in presenting a model for medium-to-small eddies. Unfortunately, the HK model is of limited use in astrophysics since it does not treat the "large eddies" region where most of the energy and other bulk properties reside. CG have recently proposed a model for LST by changing two basic ingredients of the HK model: the form of the input energy (a constant in the HK model) and, most importantly, a new "closure" that breaks the universality characterizing the HK eddies. The CG model was tested against (a) bulk properties, i.e., convective fluxes for both astrophysical and laboratory turbulence (differing some 12 orders of magnitude in the Prandtl number) and (b) spectral properties, i.e., the predicted temperature spectral function $\langle T^2(k)\rangle$ versus k fits the available data satisfactorily (Canuto and Hartke 1985). The latter test is relevant since it probes, more directly than the bulk properties, both the new closure and the use of the linear growth rates. In the present *Letter* we have used the CG model in an attempt to retrieve the generating mechanism for turbulence in MC. Since the first results are encouraging, it is important that we reiterate the assumptions and limitations underlying the model not lastly in the hope to spur interest to improve it.

We shall begin with the energy equation, equation (1) of CG. Since the nonlinear term on the right-hand side is known to have zero total integral (independently of how it is written), we have

$$\int_{k_0}^{\infty} F(k)\, n(k)\, dk = 0 \quad \text{or}$$

$$\int_{k_0}^{\infty} F(k)\left[n(k) + \nu k^2\right] dk = \nu \int_{k_0}^{\infty} k^2 F(k)\, dk. \qquad (19)$$

Let us now recall Chap. II, eq. (183) of Chandrasekhar (1961, hereafter CH), namely $\varepsilon_\nu = \varepsilon_g$, where ε_ν (rate of viscous dissipation) and ε_g (rate at which energy is released by buoyancy forces), are defined by CH equations (175) and (178). Using the expressions for u, v, w provided in CH, one can show that ε_ν is exactly the right-hand side of equation (19). Let us now compute ε_g. Let $F(k)$, $G(k)$, and $H(k)$ be the spectral function of the velocity square, temperature square, and the product of velocity and temperature. Since these quantities satisfy three coupled nonlinear differential equations which are unsolvable, one assumes that the *ratios* of any two fluctuating quantities can be approximated by the value provided by the linear analysis. Using equations (100) and (104) of CH, a long but straightforward algebra shows that ε_g is exactly the left-hand side of equation (19). The form of the first of equation (19) is therefore justified. The above procedure is general and can easily be extended to other cases. Though more laboriously, it can also be shown that equation (19) remains unchanged in the presence of rotation and magnetic fields [of course $n(k)$ becomes $n(k, \Omega, B)$]. Incidentally, the expression for the convective flux, equation (9) of CG, was obtained with this method.

Let us further remark that Ledoux, Schwarzschild, and Spiegel (1961) had already provided a derivation of equation (1) of CG from the Navier-Stokes equations and that it can also be derived from equation (19) of Yamaguchi (1963), using the form of the spectral function $H(k)$ derived above.

Having checked the integral properties of the left-hand side of equation (1) of CG, one can reinstate the integration over a finite k range and add the nonlinear transfer term, whose form is discussed in CG.

The second set of remarks concerns the form of the closure and the inclusion of other physical effects. Since the total integral of the nonlinear terms is zero, the transfer term is interpreted as a two-step process, an energy loss by eddies in the interval k_0–k [represented, as losses usually are, by $k^2F(k)$], followed by the redistribution of the same energy to all the remaining eddies in the interval k–∞ (whether this last process is viewed as due to the corrosive action of the k–∞ eddies via a turbulent viscosity does not change the physical picture). The critical problem is how to quantify this last process since it can no longer have the HK universal form. If the eddies were freely evolving (like the HK eddies), the mean free path λ_k would be approximately equal to their size. However, in the case of forced turbulence, λ_k depends on the rate at which energy is being pumped into the system, i.e., on $n(k)$. Through this dependence, one can therefore account for different physical effects, for example, compressibility [in a rotating thin disk, for example, $n(k)$ satisfies a cubic equation if $\delta\rho = 0$, and a fifth-order equation if $\delta\rho \neq 0$]. In the case of the HK eddies, where the source term is taken to be constant and the closure has a universal character, the inclusion of compressibility effects is particularly difficult to visualize and quantify, as many attempts in the literature testify. The present model for LST does permit the inclusion, through both the source and the transfer term, of physical phenomena (e.g., compressibility, rotation, magnetic fields, changes in the optical thickness, etc.), characterizing the background. What the model cannot yet do is to account for the effects of compressibility directly on the "turbulent elements" themselves. This is a limitation which future work will try to correct.

In conclusion, the goal of this *Letter* was that of introducing and illustrating a retrieval method using equation (1) as input. Since the results are interesting, it seems worthwhile to try and improve on the basic model so as to make this retrieval method more realistic.

The authors would like to thank Drs. I. Goldman, O. Hubickyj, G. Hartke, B. Elmegreen, and R. Maddalena for useful discussions on different parts of the paper. One of the authors (V. M. C.) would like to thank Dr. V. Peterson (NASA/Ames) for partial financial support.

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